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EASTERN CAPE
EDUCATION

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**COMPILATION OF:
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ALGEBRA

QUESTION 1: LIMPOPO

1.1 Solve for x :

1.1.1 $x(7 - x) = 0$ (2)

1.1.2 $3x^2 - 2x - 6 = 0$ (Correct to TWO decimal places) (3)

1.1.3 $(3 - x)(7 + x) < 0$ (3)

1.1.4 $\sqrt[3]{32} = 8^{3x} \cdot 2^{6x}$ (4)

1.1.5 $x - 4 - 2\sqrt{x - 1} = 0$ (4)

1.2 Solve the following equations simultaneously: (6)

$$x - 2y = 1 \quad \text{and} \quad x^2 - xy + 2y^2 = 2$$

1.3 It will cost R3 500 to hire a taxi for x number of friends to go on a trip. Four friends do not have money to pay. The remaining friends agree to pay extra R43,75 each to cover the cost of the taxi.

1.3.1 Calculate x , the number of friends who want to go on the trip. (5)

1.3.2 Calculate the total amount that each friend pays for this trip in order to cover the cost. (1)

[28]

QUESTION 2: MPUMALANGA

2.1 Solve for x :

2.1.1 $(3 - x)(2 - x) = 0$ (2)

2.1.2 $2x^2 + 7x = 2$ (Correct to TWO decimal places) (4)

2.1.3 $4 + 5x > 6x^2$ (4)

2.1.4 $9^x + 9 = 10 \cdot 3^x$ (4)

2.2 Solve x and y : (6)

$$y - 2x = -1 \quad \text{and} \quad y^2 - 3xy = -2$$

2.3 (5)

If $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$, calculate two values of $\frac{x}{y}$

[25]

QUESTION 3: NORTH WEST

3.1 Solve for x :

3.1.1 $x^2 = 5x$ (3)

3.1.2 $x^2 - 2x - 13 = 0$ (3)

3.1.3 $(x - 2)(1 - x) \leq 0$ (3)

3.1.4 $2\sqrt{2x - 1} = x - 11$ (5)

3.2 Solve for x and y simultaneously: (6)

$3x - y = 4$ and $x^2 + xy = 24$

3.3 Let $S = \left(1 + \frac{1}{7}\right)\left(1 + \frac{1}{8}\right)\left(1 + \frac{1}{9}\right)\left(1 + \frac{1}{10}\right) \dots \left(1 + \frac{1}{m}\right)$ where $m \in \mathbb{N}$ and $7 < m < 30$. (5)

Calculate all the value(s) of m for which S will be a natural number.
(Show ALL calculations)

[25]

QUESTION 4: GAUTENG

4.1 Solve for x :

4.1.1 $(2x - 1)^2 - 4 = 0$ (3)

4.1.2 $4x^2 - 11 = -12x$ (Correct to TWO decimal places) (3)

4.1.3 $15x - 4 < 9x^2$ (4)

4.1.4 $\sqrt{2x - 2} - \sqrt{7 - 2x} = 1$ (5)

4.2 Solve the following equations simultaneously: (5)

$a^2b^2 - 2ab - 8 = 0$ and $\log_2(a + 5) = 3$

4.3 If $p = \frac{\sqrt{x + 2}}{\sqrt{16 - x^2}}$, for which values of x will p be real? (4)

[24]

QUESTION 5: FREE STATE

5.1 Solve for x :

5.1.1 $(x - 1)(2x - 1) = 0$ (2)

5.1.2 $(x - 1)(2x + 1) = 4$ (Correct to TWO decimal place) (4)

5.1.3 $x + \sqrt{x - 2} = 4$ (5)

5.1.4 $3x^2 + x \leq 0$ (3)

5.2 Solve for x and y in the following simultaneous equations: (6)

$xy = 8$ and $2x + y = 17$

5.3 Simplify the following WITHOUT USING A CALCULATOR: (3)

$$\sqrt{\sqrt{21x^2} - \sqrt{5x^2}} \times \sqrt{\sqrt{21x^2} + \sqrt{5x^2}}$$

[23]

QUESTION 6: EASTERN CAPE

6.1 Solve for x :

6.1.1 $x^2 + x - 30 = 0$ (3)

6.1.2 $x(2x - 6) = -3$ (Correct to TWO decimal places) (4)

6.1.3 $x^2 - 2x + 1 > 0$ (3)

6.1.4 $2x - 1 = \sqrt{4 - 5x}$ (4)

6.2 Solve simultaneously for x and y : (6)

$y - 2x = -1$ and $2y^2 + 4xy = 6x^2$

6.3 Given the quadratic equation: $2x^2 - px + 1 = 0$, $x \in \mathbb{R}$. (5)

Determine the possible value(s) of p , such that the equation has two **unequal** real roots.

[25]

QUESTION 7: KWA-ZULU NATAL

7.1 Solve for x :

7.1.1 $x(x - 4) = 0$ (2)

7.1.2 $2x^2 + 3x = 7$ (Write your answer to TWO decimal places) (4)

7.1.3 $x^2 - 5x + 4 > 0$ (3)

7.1.4 $3^{2x} - 10 \cdot 3^x + 21 = 0$ (5)

7.2 Solve simultaneously for x and y : (6)

$x + y = 2$ and $x^2 + y^2 + 6x = 4y - 4$

7.3 The roots of a quadratic equation: $x = \frac{-4 \pm \sqrt{25 - n^2}}{6}$ (3)

For which values of n will the roots be equal?

[23]

SEQUENCES & SERIES

QUESTION 1: LIMPOPO

- 1.1 Given the arithmetic sequence: 14 ; 21 ; 28 ; ... ; 336.
- 1.1.1 How many terms are there in the sequence? (3)
- 1.1.2 Calculate the sum of all the terms of the sequence. (2)
- 1.1.3 The sum of two consecutive terms of the sequence: 21 ; 35 ; 49 ; ... is 308. Determine the two terms. (4)
- 1.2 Show that $T_n = 4n^2 + n$ is the general term of a quadratic sequence with the following properties: (4)
- 68 is the fourth term of the quadratic sequence.
 - 21 and 29 are the second and third terms, respectively, of the sequence of the first differences of the quadratic sequence.

[13]

QUESTION 2: LIMPOPO

- 2.1 The sum to infinity of a geometric series is 6, and the sum to infinity of the squares of the terms of the series is 6.
- 2.1.1 Write down the second term of the series of squares in terms of a and r . (1)
- 2.1.2 Calculate the values of a and r . (6)
- 2.1.3 Write down T_6 of the series of squares. (2)
- 2.2 There are 50 litres of water in a container. Water evaporates from this container at the rate of 1% per hour under the certain weather conditions. How much water will be left in the container after 8 hours if left under these weather conditions? (3)

[12]

QUESTION 3: MPUMALANGA

3.1 Consider the following quadratic sequence:

$$319 ; 280 ; 243 ; 208 ; 175 ; 144$$

3.1.1 Show that the n^{th} term of the sequence is $T_n = n^2 - 42n + 360$. (4)

3.1.2 Determine which term has a value of 0? (3)

3.1.3 Which term in the sequence will have the lowest value? (2)

3.2 $3t ; 4t - 1 ; 23$ are the first three terms of an arithmetic sequence.

3.2.1 Prove that $t = 5$ (2)

3.2.2 Calculate the sum of the first 50 terms of the series. (4)

[15]

QUESTION 4: MPUMALANGA

4.1 The following geometric series is given:

$$4 + 12 + 36 + \dots \text{ to 15 terms}$$

4.1.1 Write down the series in sigma notation. (2)

4.1.2 Calculate the series. (2)

4.1.3 Is this series convergent? Provide a reason for your answer. (2)

4.2 The sum of an infinite geometric series is 13,5 ($r \neq 1$)

The sum of the same series, calculated from the third term is 1,5.

4.2.1 Calculate r if $r > 0$. (4)

4.2.2 Hence, determine the first THREE terms of the series. (2)

[12]

QUESTION 5: NORTH WEST

- 5.1 Given the quadratic number pattern: $-120 ; -99 ; -80 ; -63 ; \dots$
- 5.1.1 Write down the next TWO terms of the pattern. (2)
- 5.1.2 Determine the n^{th} term of the number pattern in the form $T_n = an^2 + bn + c$. (4)
- 5.1.3 What value must be added to T_n for the sequence to have only one value of n for which $T_n = 0$? (4)
- 5.2 Given the finite arithmetic series: $9 + 14 + 19 + \dots + 124$
- 5.2.1 Determine the general term of this series in the form $T_n = dn + c$ (2)
- 5.2.2 Write the series in sigma notation. (3)
- 5.3 Given the arithmetic series: $2^x + 2^{x+1} + 3 \cdot 2^x + 2^{x+2} + \dots$ (4)
- Calculate the value of x if the sum of the first 30 terms add up to 3 720. [19]

QUESTION 6: NORTH WEST

- 6.1 Given: $5 ; 10 ; 20 ; \dots$ a geometric sequence.
- 6.1.1 Determine the n^{th} term. (1)
- 6.1.2 Calculate the sum of the first 18 terms. (2)
- 6.2 The first term and second terms of a geometric series are given as $(2x - 4)$ and $(4x^2 - 16)$ respectively. Determine the value(s) of x for which the series will converge. (4)
- 6.3 A convergent geometric series has a first term of 2 and $r = \frac{1}{\sqrt{2}}$. (3)
- Calculate $\frac{S_\infty}{S_2}$
- [10]

QUESTION 7: GAUTENG

7.1 The following is an arithmetic sequence:

$$1 - p ; 2p - 3 ; p + 5 ; \dots$$

7.1.1 Calculate the value p . (2)

7.1.2 Write down the value of:

(a) The first term of the sequence (1)

(b) The common difference (1)

7.1.3 Explain why NONE of the number in this arithmetic sequence are perfect squares. (2)

7.2 The following sequence of numbers forms a quadratic sequence:

$$-3 ; -2 ; -3 ; -6 ; -11 ; \dots$$

7.2.1 The FIRST differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

7.2.2 Calculate the first difference between 35th and 36th terms of the quadratic sequence. (1)

7.2.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

7.2.4 Show that the sequence of numbers will NEVER contain a positive term. (2)

[16]

QUESTION 8: GAUTENG

8.1 Given: $S_n = 4n^2 + 1$. Determine T_6 . (3)

8.2 For which values of x will the following series converge? (3)

$$(4x - 3) + (4x - 3)^2 + (4x - 3)^3 + \dots$$

8.3 Calculate: $\sum_{k=3}^5 (-1)^k \cdot \frac{2}{k}$ (2)

[8]

QUESTION 9: FREE STATE

9.1 Consider the series: $a + (a + d) + (a + 2d) + \dots$

9.1.1 Prove that the sum of the first n terms of this arithmetic series will be: (3)

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

9.1.2 Given: $2^x + 2 \cdot 2^x + 3 \cdot 2^x + \dots$. The sum of the first 20 terms of the series is 1 680. Calculate the value of x . (4)

9.2 Given: $S_n = \frac{n^2 + n}{4}$, calculate T_8 . (3)

9.3 Consider the series: $32 + (-16) + 8 + (-4) + \dots$
Calculate the sum of the first 10 terms of the series. (3)

[13]

QUESTION 9: FREE STATE

10.1 Given the quadratic number pattern: $-4; -6; -10; -16; \dots$

10.1.1 Determine T_n (4)

10.1.2 Between which consecutive terms of the patterns is the difference -100 ? (4)

10.2 Given that: $\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n = -\frac{5}{3}$. Calculate the value of k . (4)

[12]

QUESTION 11: EASTERN CAPE

11.1 The tenth and the seventh terms of an arithmetic sequence are 21 and 49 respectively.

11.1.1 Determine the common difference of the sequence. (3)

11.1.2 Calculate: $T_1 + T_{18}$ (3)

11.2 Given: $\sum_{n=1}^{\infty} (4n - 19) = 1\,189$

11.2.1 Write down the first three terms of the series. (1)

- 11.2.2 Calculate the value of m . (4)
- 11.3 $-78; -76; -72; -66; \dots$ is a quadratic number pattern.
- 11.3.1 Write down the next two terms of the number pattern. (1)
- 11.3.2 Determine the n^{th} term of the number pattern in the form, (4)
 $T_n = an^2 + bn + c$.
- 11.3.3 A constant k is added to T_n such that all the terms of the quadratic number pattern become positive. Determine the value(s) of k . (2)

[18]

QUESTION 12: EASTERN CAPE

- 12.1 The first term of a geometric sequence is 81 and the common ratio is r . The sum of the first and third terms of the same geometric sequence is 117. Calculate the value of r . (4)
- 12.2 Given the convergent geometric series: $3^x + 9^x + 27^x + 81^x + \dots$
- 12.2.1 Write down common ratio in terms of x (1)
- 12.2.2 Calculate the value x , if $S_\infty = \frac{1}{2}$ (3)

[8]

QUESTION 13: KWA-ZULU NATAL

The values below are the consecutive terms of a quadratic sequence.
 The fourth term is 49.

$- ; - ; - ; 49 ; 77 ; 111 ; 151 ; \dots \dots \dots$

- 13.1 Determine the third term of the quadratic sequence. (1)
- 13.2 Determine the general term, T_n of the quadratic sequence. (4)
- 13.3 Between which two consecutive terms of the quadratic sequence is the first difference 418. (3)

[8]

QUESTION 14: KWA-ZULU NATAL

- 14.1 The first two terms of a geometric sequence are x and $(x + 1)$.
- 14.1.1 Write down the common ratio. (1)
- 14.1.2 Write down the third term. (2)
- 14.1.3 If $x = 2$, will the sequence converge? Motivate your answer. (2)
- 14.2 The given sequence below has **four terms only**, such that the first three terms, T_1 ; T_2 and T_3 form an arithmetic sequence and the last three terms, T_2 ; T_3 and T_4 form a geometric sequence. (5)

$$6 ; a ; b ; 16$$

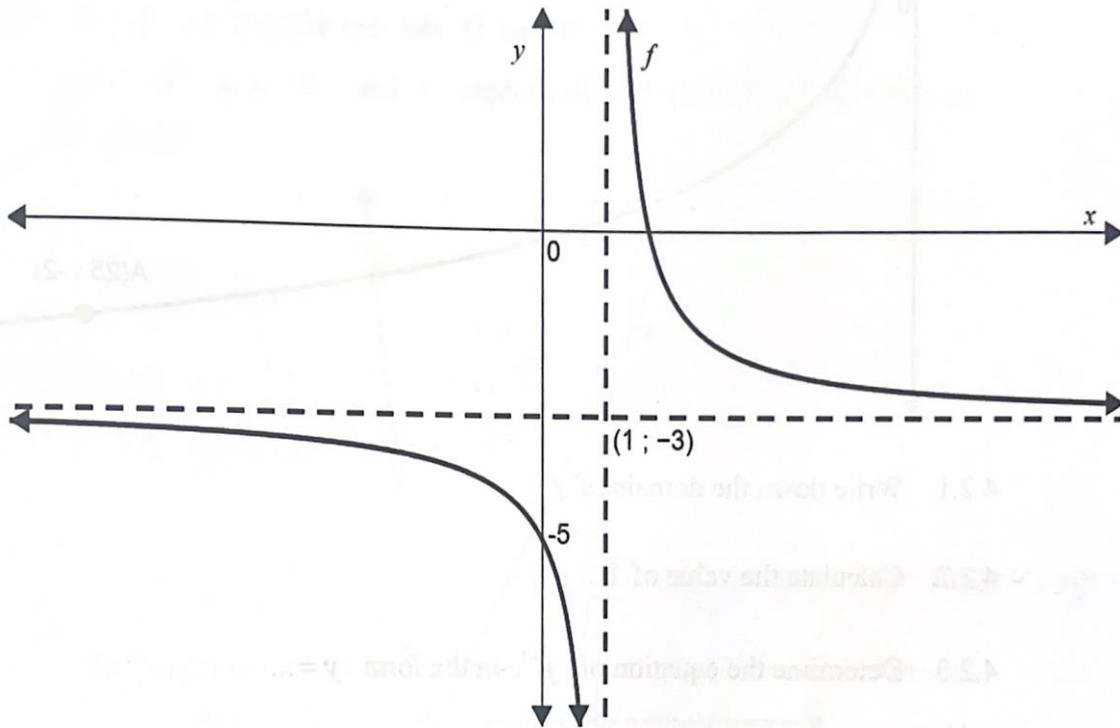
Calculate the values of a and b .

[10]

FUNCTIONS

QUESTION 1: LIMPOPO

The graph of $f(x) = \frac{a}{x+p} + q$ is sketched below. The asymptotes of f intersect at $(1; -3)$. The y -intercept of f is $(0; -5)$.

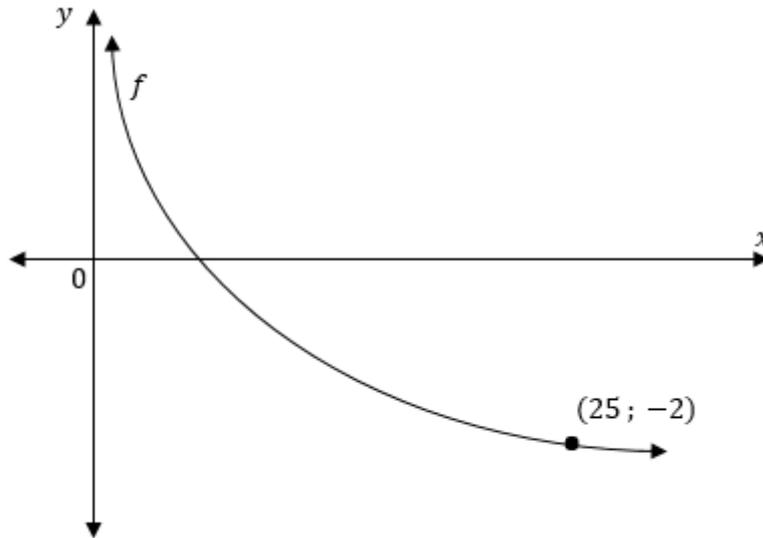


- 1.1 Determine the values p and q . (2)
- 1.2 Calculate the value of a . (3)
- 1.3 If $f(x) = \frac{2}{x-1} - 3$ is translated to g such that $g(x) = \frac{2}{x-3} + 3$, (2)
describe the transformation of f to g .
- 1.4 Write down the equation of the horizontal asymptote of g . (1)

[8]

QUESTION 2: LIMPOPO

The graph of $f(x) = \log_b x$ is sketched below. $A(25; -2)$ is a point on f .



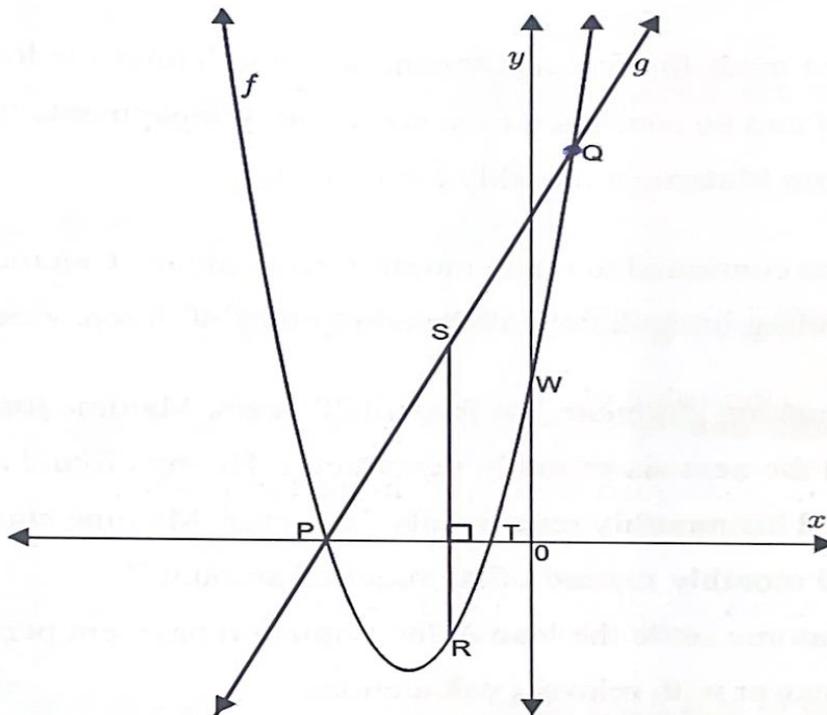
- 2.1 Write down the domain of f . (1)
- 2.2 Calculate the value of b . (2)
- 2.3 Determine the equation of f^{-1} in the form $y = \dots$ (2)
- 2.4 For which values of x is $\left(\frac{1}{5}\right)^{x+3} - 5 < 20$? (2)

[7]

QUESTION 3: LIMPOPO

The graphs of $f(x) = ax^2 + bx + c$ and $g(x) = 2x + 10$ are sketched below.

- Graph f intersects the x -axis at $P(-5; 0)$ and $T(-1; 0)$, and the y -axis at $W(0; 5)$.
- The two graphs intersect at points Q and P .
- R and S are points on f and g respectively such that SR is perpendicular to the x -axis.

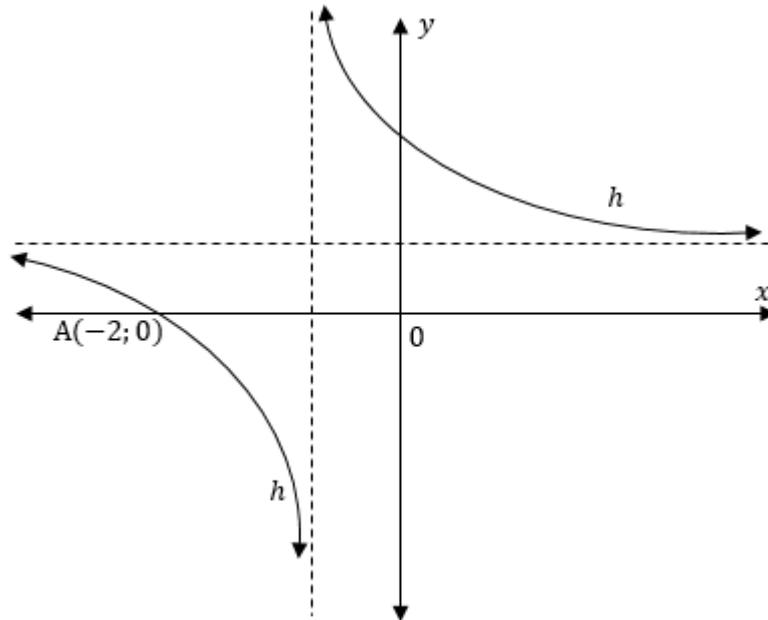


- 3.1 Show that $f(x) = x^2 + 6x + 5$ (3)
- 3.2 Calculate the coordinates of Q . (4)
- 3.3 Show that $f(x) \neq -5$ for all values of x . (3)
- 3.4 Consider point R when SR is at its maximum in the interval $x_P < x < x_Q$.
Determine:
- 3.4.1 The gradient of the tangent to f at R . (4)
- 3.4.2 The equation of the tangent to f at R . (3)
- 3.5 Consider $x > x_P$, for which values of x is $g(x) - g^{-1}(x) > 15$? (3)

[20]

QUESTION 4: MPUMALANGA

In the diagram, $A(-2; 0)$ is the x -intercept of $h(x) = \frac{a}{x+1} + 2$.

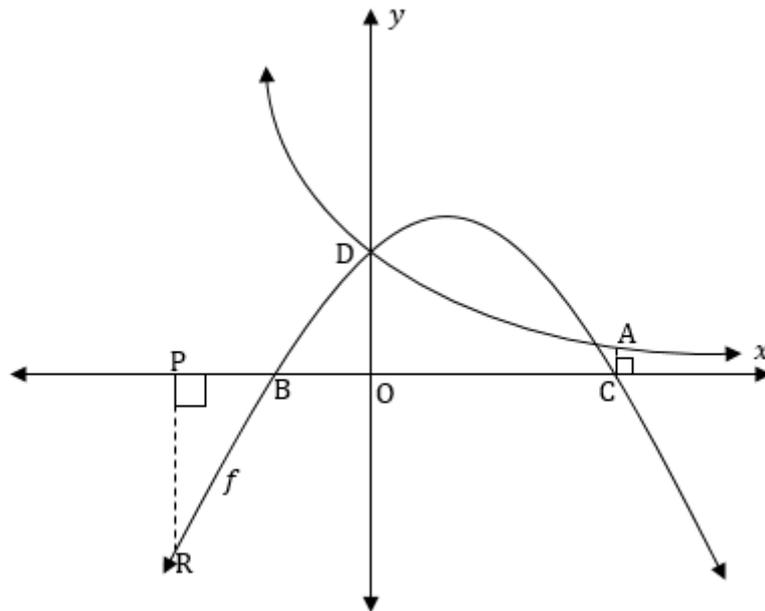


- 4.1 Calculate a . (2)
- 4.2 Calculate the average gradient of h between $x = -4$ and $x = -2$. (3)
- 4.3 For which values of x is $h(x) > 4$? (2)
- 4.4 Determine the equation of the axis of symmetry of h that has a positive gradient. Leave your answer in the form $y = \dots$ (2)
- 4.5 Write down the equations of the asymptotes of g , if g is the reflection of h over the x -axis. (2)

[11]

QUESTION 5: MPUMALANGA

Sketched below is $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + k$ and $h(x) = q^x$. The x -intercepts of A and B. h and f intersect the y -axis at D. A is a point on h vertically above with y -value of $\frac{1}{4}$. PR is perpendicular to the x -axis.



- 5.1 Calculate the coordinates of D. (1)
- 5.2 Write down the value of k . (1)
- 5.3 Calculate the length of BC. (4)
- 5.4 Calculate the length of OR if OP is 2 units. (2)
- 5.5 Consider $h(x) = q^x$
 - 5.5.1 Calculate the value q . (3)
 - 5.5.2 Hence, determine the equation of h^{-1} , in the form $y = \dots$ (2)
- 5.6 Write down the range of h . (1)
- 5.7 Determine the equation of g , the tangent to f at B. (5)
- 5.8 For which values of x is $x \cdot f'(x) < 0$? (2)

[21]

QUESTION 6: NORTH WEST

Given: $f(x) = \frac{2}{x-1} - 2$

- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Draw a graph of f . Clearly label ALL the intercepts with the axes and the asymptotes on your graph. (4)
- 4.3 Determine the equation of the line of symmetry of f for $m < 0$. (2)
- 4.4 For which values of x will $f(x) \leq -4$? (2)

[10]

QUESTION 7: NORTH WEST

The graph of $g(x) = a\left(\frac{1}{5}\right)^x - 5$ passes through $A(-2; -4)$.

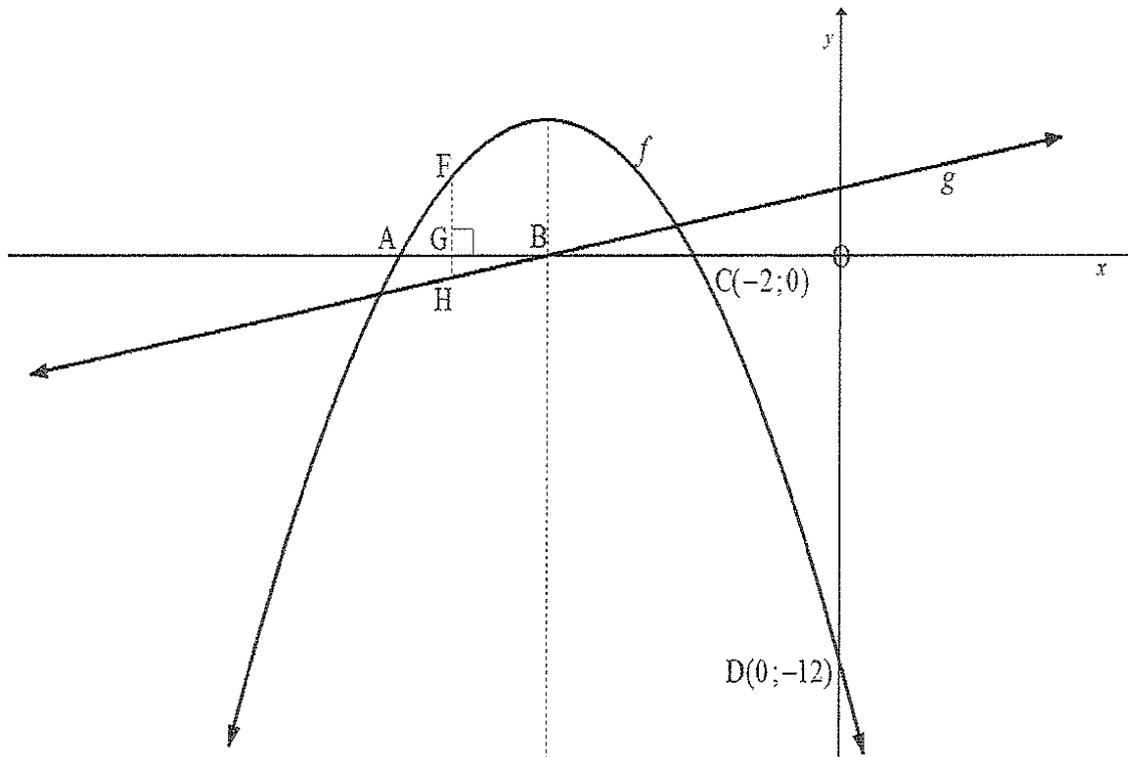
- 7.1 Show that $a = \frac{1}{25}$ (2)
- 7.2 Determine the coordinates of the x -intercept of g . (3)
- 7.3 Given: $h(x) = \left(\frac{1}{5}\right)^x$
- 7.3.1 Determine the equation of $x^{-1}(x)$, the inverse of h , in the form $y = \dots$ (2)
- 7.3.2 Describe the transformation from g to h . (2)

[9]

QUESTION 8: NORTH WEST

The sketch below represents the following graphs: $f(x) = ax^2 + bx + c$ and $g(x) = \frac{1}{2}x + 2$

- The line of symmetry of f cuts the x -axis at B, which is also the x -intercept of g .
- $C(-2; 0)$ is an x -intercept of f .
- $D(0; -12)$ is an y -intercept of f .



- 8.1 Calculate the coordinate of B. (2)
- 8.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$ (4)
- 8.3 If $f(x) = -x^2 - 8x - 12$, determine the coordinates of G if FH, the vertical length between the points of intersection of f and g , is a maximum. (4)
- 8.4 For which values of x will $x \cdot f'(x) > 0$? (2)

[12]

QUESTION 9: GAUTENG

Consider the graphs of $g(x) = \frac{6}{x+3} + \frac{3}{2}$ and $h(x) = \frac{6}{x-3} + 2$

- 9.1 Write down the domain of g . (1)
- 9.2 Write down the range of g . (1)
- 9.3 If the graph of g is shifted so that it coincides with the graph of h .
- 9.3.1 How many units must the graph be shifted horizontally? (1)
- 9.3.2 How many units must the graph be shifted vertically? (1)
- 9.4 Write down the equations of the asymptotes of g . (2)
- 9.5 Calculate the x -intercept of g . (1)
- 9.6 Sketch the graph of g in your ANSWER BOOK. Show clearly all the asymptotes and intercepts with the axes. (3)
- 9.7 Determine the value of k if $h(x) = -x + k$ is an axis of symmetry of g . (2)
- 9.8 For which value(s) of x will $\frac{6}{x+3} - \frac{3}{2} > -x + k$? (1)
- 9.9 The graph of g is reflected in the x -axis. Write down the new equation in the form $y = \dots$ (1)

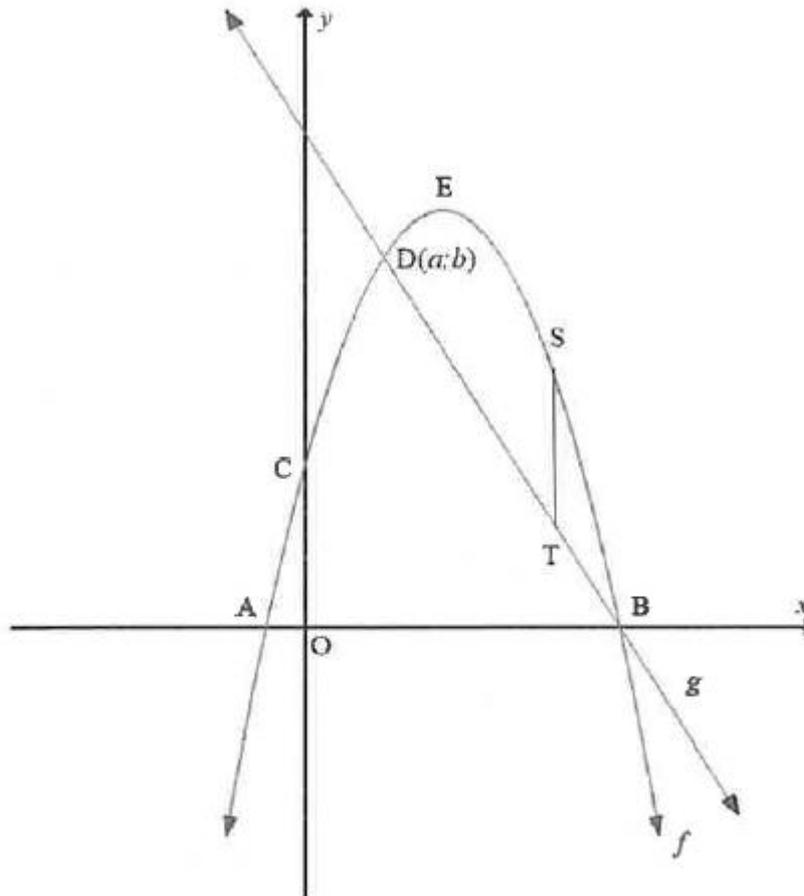
[14]

QUESTION 10: GAUTENG

The graphs of $f(x) = -\left(x - \frac{7}{3}\right)^2 + \frac{81}{4}$ and $g(x) = -3x + 24$ are sketched below.

The graphs of f and g intersect at points D and B.

Points A and B are the x -intercepts of f .

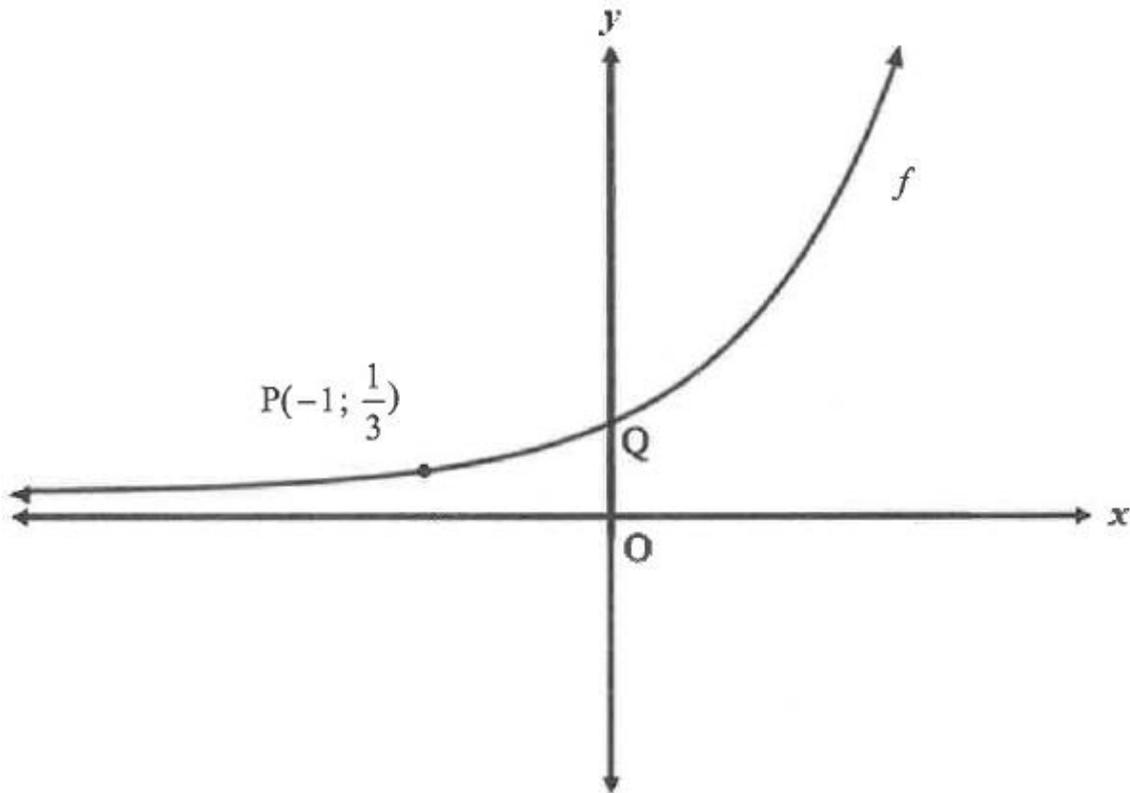


- 10.1 Write down the coordinates of E, the turning point of f . (1)
- 10.2 Determine the average gradient of the curve of f between $x = 1$ and $x = 5$. (4)
- 10.3 Determine the value of a , the x -coordinate of point D. (3)
- 10.4 Point $S(x; y)$ is a point on the graph of f , where $a \leq x \leq 8$. (2)
Line ST is drawn parallel to the y -axis with point T on the graph of g .
Determine ST in terms of x .
- 10.5 Calculate the maximum length of ST . (3)

[13]

QUESTION 11: GAUTENG

The graph of $f(x) = 3^x$ is sketched below. $P(-1; \frac{1}{3})$ is a point on f .



- 11.1 Write f^{-1} in the form $y = \dots$ (1)
- 11.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x - 2)$ on the same set of axes in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes. (4)
- 11.3 Use your graphs drawn in QUESTION 11.2 to solve for x if $\log_3(x - 2) < 1$ (2)

[7]

QUESTION 12: FREE STATE

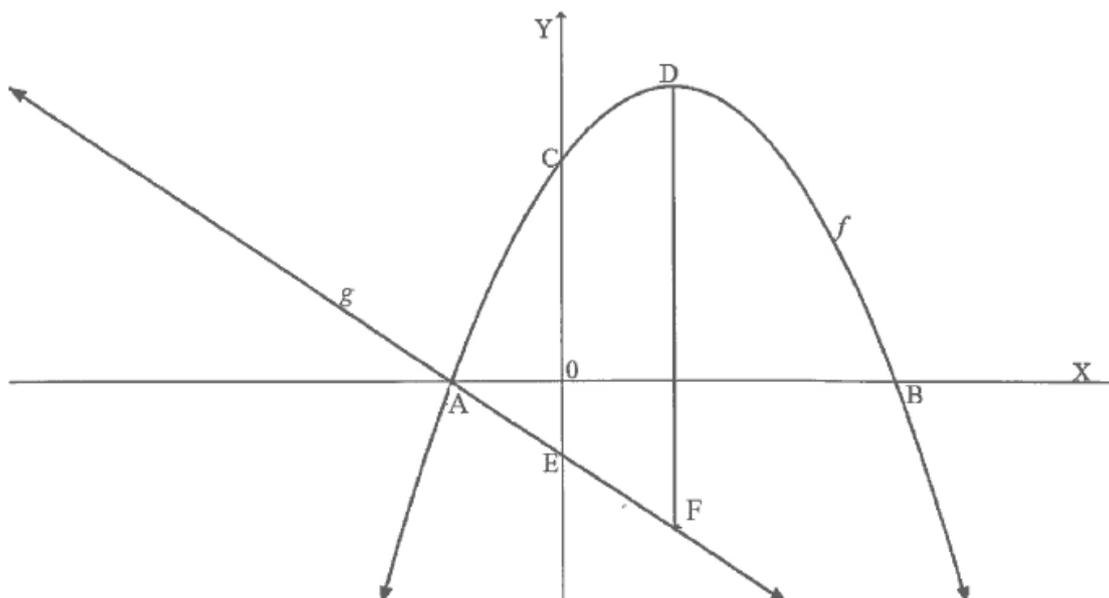
Given the function: $h(x) = -\frac{2}{x-2} + 2$

- 12.1 Write down the equations of the asymptotes of h . (2)
- 12.2 Calculate the x -intercept of h . (2)
- 12.3 Draw the graph of h . Clearly show all asymptotes and intercepts with the axes. (4)
- 12.4 Determine the equation of the axis of symmetry of h , in the form $y = mx + c$, where $m < 0$. (3)
- 12.5 Determine the range of $-h(x) + 1$. (2)
- 12.6 Determine the values of x where $h(x) \leq 0$. (2)

[15]

QUESTION 13: FREE STATE

In the diagram, the graphs of $f(x) = -\frac{1}{2}x^2 + 2x + 6$ and $g(x) = -x - 2$ are drawn. C and E are the y -intercepts of f and g , respectively. The parabola has a turning point at D and cuts the x -axis at A and B. A is also the x -intercept of g . DF is a line parallel to the y -axis with F a point on g .



- 13.1 Calculate the:
- 13.1.1 Coordinates of D. (3)

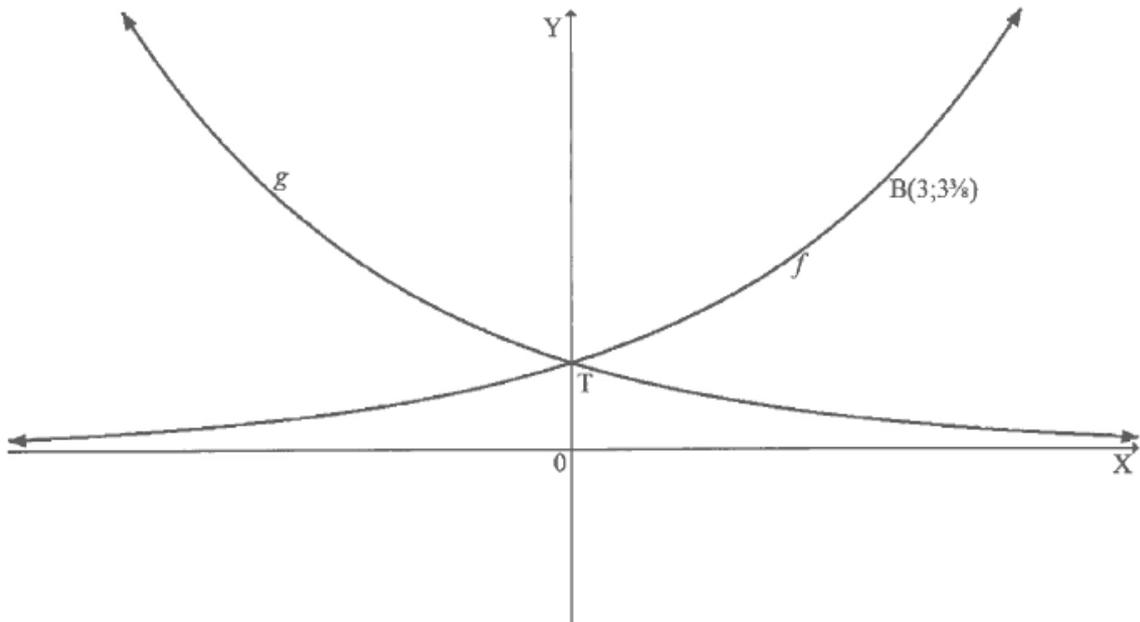
- 13.1.2 Distance DF. (3)
- 13.2 For which values of k will $f(x) = k$ have two positive roots? (2)
- 13.3 Given that $f(x) = h'(x)$. Determine the x -coordinate of the turning points of h . (3)
- 13.4 Determine the value(s) of x where $f'(x) \times g(x) \leq 0$. (2)

[13]

QUESTION 14: FREE STATE

The diagram show the graphs of $f(x) = a^x$ and g , the reflection of f in the y -axis.

$B\left(3; 3\frac{3}{8}\right)$ lies on f . The two graphs intersect at T .



- 14.1 Write down the coordinates of T . (1)
- 14.2 Calculate the value of a . (2)
- 14.3 Determine the equation of g . (2)
- 14.4 Write down the equation of $f^{-1}(x)$, the inverse of f , in the form $y = \dots$ (2)
- 14.5 For which values of x will $f^{-1}(x) \leq 1$? (3)

[10]

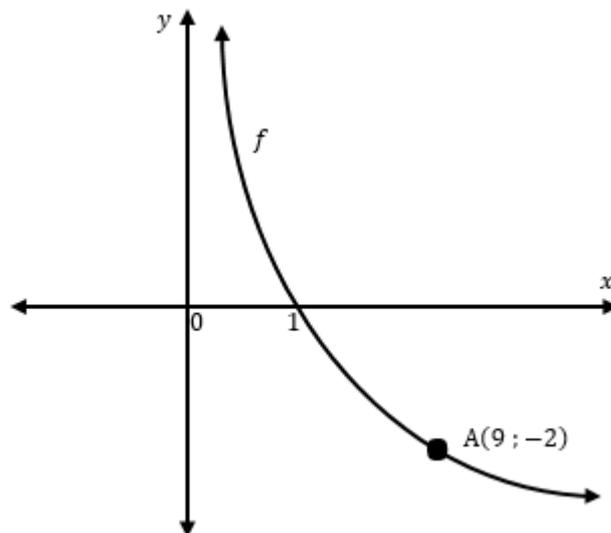
QUESTION 15: EASTERN CAPE

Given: $f(x) = \frac{2}{x-5} + 3$

- 15.1 Write down the equations of the asymptotes of f . (2)
- 15.2 Write down the range of f . (1)
- 15.3 Determine the coordinates of the x -intercept and y -intercept of f . (3)
- 15.4 Sketch the graph of f , clearly showing all asymptotes and intercepts with the axes. (4)
- 15.5 Describe the transformation that the graph of f has to undergo to for the graph of h , where $h(x) = -\frac{2}{x-5} - 5$ (3)

[13]**QUESTION 16: EASTERN CAPE**

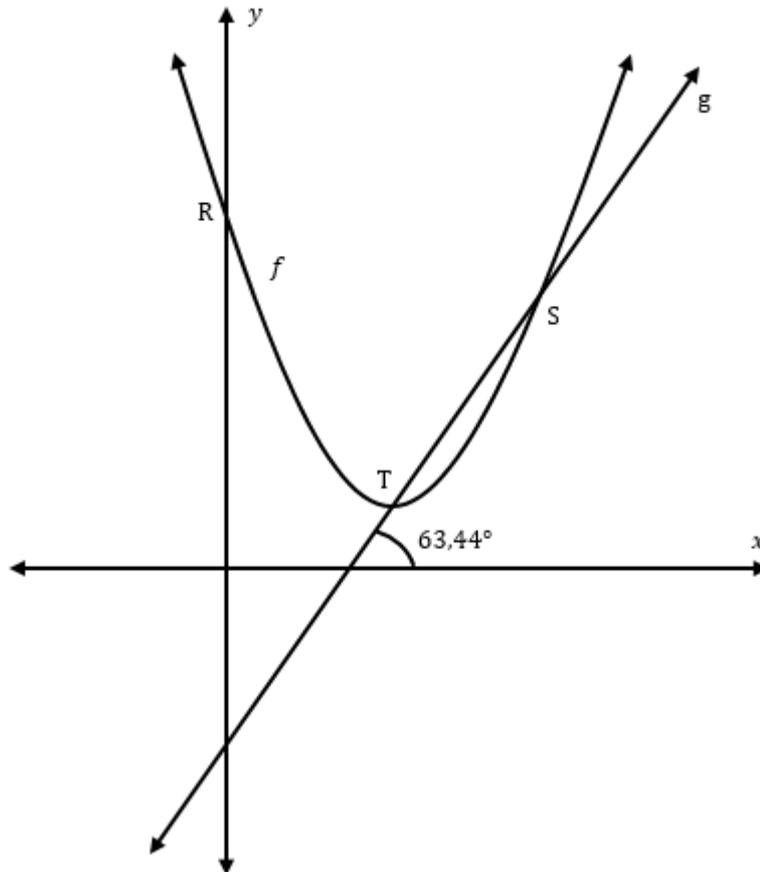
The diagram below shows the graph of $f(x) = \log_b x$, where b is a constant. f passes through the point $A(9; -2)$.



- 16.1 Show that $b = \frac{1}{3}$ (2)
- 16.2 Determine the equation of f^{-1} , the inverse f , in the form $y = \dots$ (2)
- 16.3 For which values of x if $f(x) \geq 0$. (2)
- 16.4 Write down the equation of the asymptote of g , if $g(x) = f^{-1}(x + 1)$ (2)

QUESTION 17: EASTERN CAPE

The diagram below shows the graphs of $f(x) = x^2 - 6x + 11$ and $g(x) = ax + b$. The graphs of f and g intersect at S and T, where T is the turning point of f . The angle of inclination of g is $63,44^\circ$.



- 17.1 Calculate the coordinates of T. (4)
- 17.2 Determine the equation of g in the form $y = mx + c$. (3)
- 17.3 Hence, or otherwise determine the coordinate of S. (4)
- 17.4 Determine the values of:
- 17.4.1 x , for which $f(x) \leq 6$ (2)
- 17.4.2 k , for which $f(x) + k$ will have real roots (2)

[15]

QUESTION 18: KWA-ZULU NATAL

Given: $g(x) = \frac{-3}{3-x} - 1$

- 18.1 Write down the equations of the asymptotes of g . (2)
- 18.2 Calculate the x - and y -intercepts of g . (3)
- 18.3 Draw the graph of g showing all asymptotes and intercepts with the axes. (3)
- 18.4 Determine the equation of the axis of symmetry of g with the negative gradient. (2)

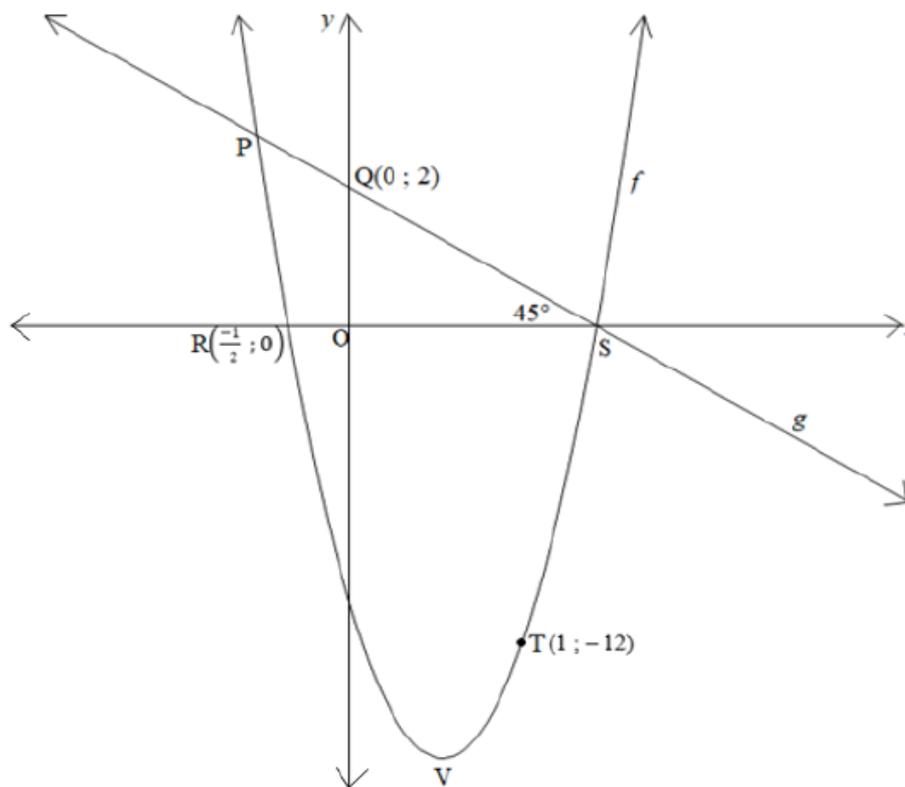
[10]

QUESTION 19: KWA-ZULU NATAL

The graphs of a parabola f and a straight line g intersect at P and S as shown below.

Points $R\left(-\frac{1}{2}; 0\right)$ and $T(1; -12)$ are on the parabola, and $Q(0; 2)$ is a point on g .

V is the turning point of the parabola. The straight line makes an angle of 45° with the x -axis.



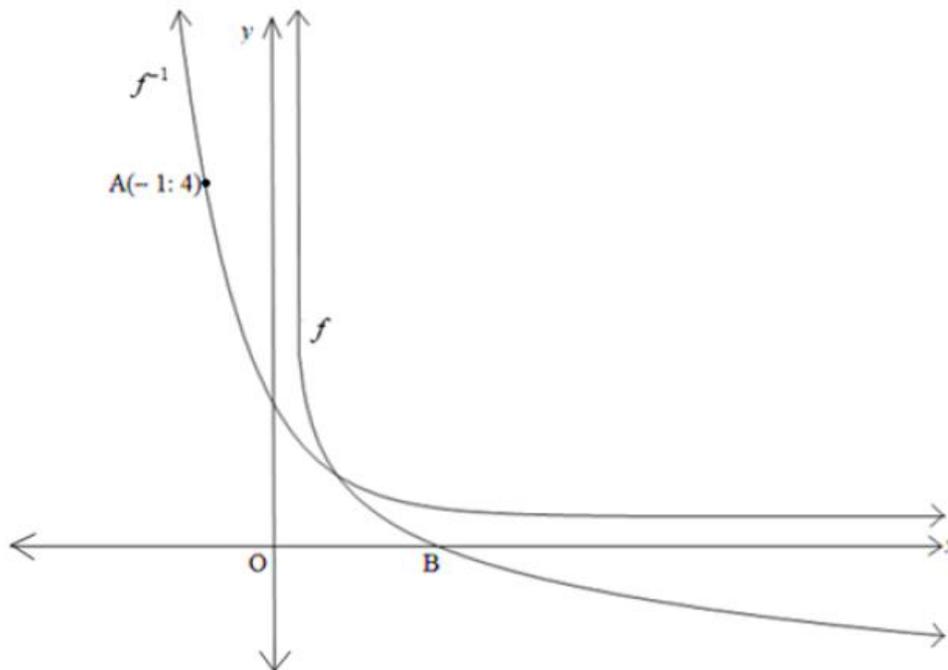
- 19.1 Show that the equation of g is $g(x) = -x + 2$. (2)

- 19.2 Write down the coordinates of S. (2)
- 19.3 Show that the equation of f is $f(x) = 8x^2 - 12x - 8$. (4)
- 19.4 Determine the coordinates of V the turning point of f , (3)
- 19.5 Use the graph to determine the values of k , if $f(x) = k$ has two unequal roots. (1)
- 19.6 For which values x if $f'(x) \cdot g'(x) < 0$? (1)
- 19.7 A new graph h is obtained by reflecting the graph of f about the y -axis and then shifting it in the upward direction so that the x -axis is the tangent of the new graph h . Write down the coordinates of point T' (the image of T) which lies on the graph of h . (2)

[15]

QUESTION 20: KWA-ZULU NATAL

The diagram shows the graphs of $f(x) = \log_a x$ and f^{-1} . Point $A(-1; 4)$ lies on the inverse of f .



- 20.1 Determine the coordinates of A' , the image of A after reflection in the line $y = x$. (1)
- 20.2 Show that the value of a is $\frac{1}{4}$. (2)

20.3 Determine the equations of f^{-1} in the form $y = \dots$ (1)

20.4 Calculate the length of AB. (3)

20.5 Write down the value(s) of x for which $f(x) < -1$. (1)

[8]

DIFFERENTIAL CALCULUS

Part 1: Differentiation

QUESTION 1: LIMPOPO

1.1 If $f(x) = 4x^2 - x$, determine $f'(x)$ from the first principles. (5)

1.2 Determine:

1.2.1 $\frac{dy}{dx}$ if $y = -x^3 - 16x^2 + 6x$ (3)

1.2.2 $f'(x)$ if $f(x) = \frac{4}{5x^{-3}} - \frac{3}{\sqrt[3]{x^2}}$ (4)

[12]

QUESTION 2: MPUMALANGA

2.1 Given: $f(x) = -2x^2 + 1$ (5)

Determine $f'(x)$ from first principles.

2.2 Determine:

2.2.1 $f'(x)$ if $f(x) = \frac{1}{2}x^2 - \frac{5}{x}$ (3)

2.2.2 $D_x \left[\frac{-2x^2 + \sqrt[4]{x}}{x^2} \right]$ (3)

[11]

QUESTION 3: NORTH WEST

3.1 Determine $f'(x)$ from the first principles if $f(x) = 1 - x^2$ (5)

3.2 Determine:

3.2.1 $D_x \left[3x^2 - \frac{2}{x} \right]$ (3)

3.2.2 $\frac{dy}{dx}$ if $y = \sqrt{x}(\sqrt[3]{x} - 5x)$ (4)

[12]

QUESTION 4: GAUTENG

4.1 Determine $f'(x)$ from the first principles if $f(x) = 3x^2 - 6$ (4)

4.2 Determine the derivative of $f(x) = \left(2\sqrt{x} - \frac{1}{x}\right)^2$. (5)

4.3 Given: $f(x) = 3x^3 - 3x^2 + 6x - 2$ (4)

Determine the interval for which f is concave up.

[13]

QUESTION 5: FREE STATE

5.1 Determine the derivative of $f(x) = 3 - x^2$ using FIRST PRINCIPLES. (5)

5.2 Determine:

5.2.1 $D_x \left[\frac{2}{x} - \sqrt{x} \right]$ (4)

5.2.2 $\frac{dy}{dx}$ if $y = (x^3 - 1)^2$ (3)

[12]

QUESTION 6: EASTERN CAPE

6.1 Determine $f'(x)$ from the first principles if $f(x) = 1 - x^2$ (5)

6.2 Determine:

6.2.1 $D_x \left(x - \frac{1}{x} \right)^2$ (3)

6.2.2 $\frac{dy}{dx}$ if $y = \frac{x^5}{10} - \frac{2}{\sqrt{x}}$ (3)

[11]

QUESTION 7: KWA-ZULU NATAL

7.1 From first principles, determine the derivative of $f(x) = 2x^2 + 9$. (5)

7.2 Determine the following:

7.2.1 $\frac{dy}{dx}$ if $y = x(2x + 1)$ (2)

7.2.2 $\frac{dy}{dx}$ if $\sqrt{y + x} = x + 3$ (3)

7.2.3 $\frac{d}{dx} \left[\frac{4 + \sqrt{3x}}{x} \right]$ (3)

7.3 The derivative of a function f at point P is given below:

$$\lim_{h \rightarrow 0} \frac{[(2 + h)^3 + 1] - [2^3 + 1]}{h} = 12$$

7.3.1 Write down the equation of f . (2)

7.3.2 A tangent is drawn at P. Determine the equation of the tangent to f at point P. (3)

[18]

Part 2: Cubic functions

QUESTION 1: LIMPOPO

Given: $f(x) = ax^3 + bx^2 + cx + d$ with the following properties:

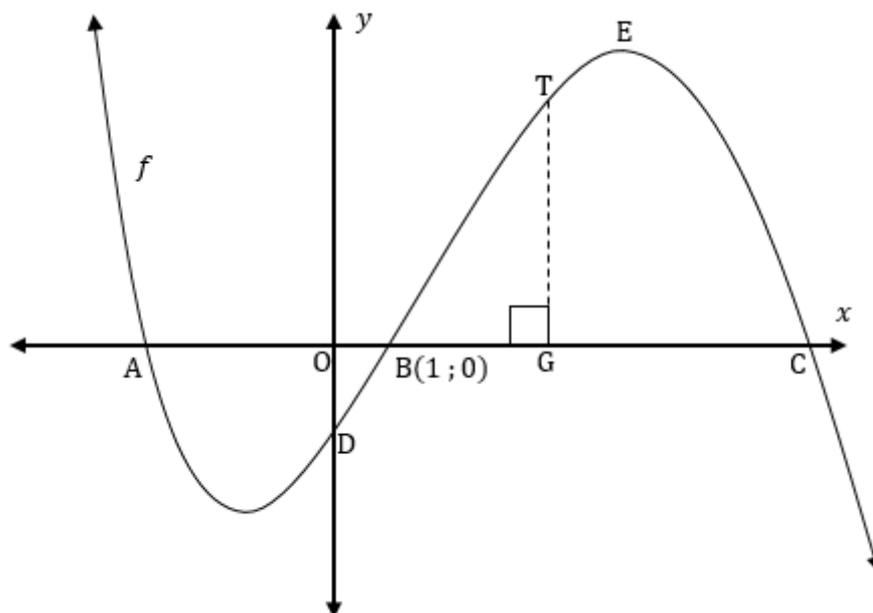
- $f(-2) = f(1) = 0$
- $f'(1) = f'(-1) = 0$
- $f(-1) = 4$
- $f(0) = 2$
- $f'(x) < 0$ for $-1 < x < 1$

- 1.1 Draw the graph of f . Indicate clearly the coordinates of the turning points and the intercepts. (4)
- 1.2 Determine the equation of f in the form $y = \dots$ (5)
- 1.3 If $g(x) = f(x - 3)$, write down the x -intercepts of g . (2)
- 1.4 Determine the values of p for which $x^2 - 3x + 2 = p$ will have exactly one root. (2)

[13]

QUESTION 2: MPUMALANGA

In the diagram, the graph of $f(x) = -x^3 + 5x^2 + 8x - 12$ is drawn. A, B and C are the x -intercepts of f . T is a point of f and G is a point on the x -axis such that TG is perpendicular to the x -axis. D is the y -intercept of f .

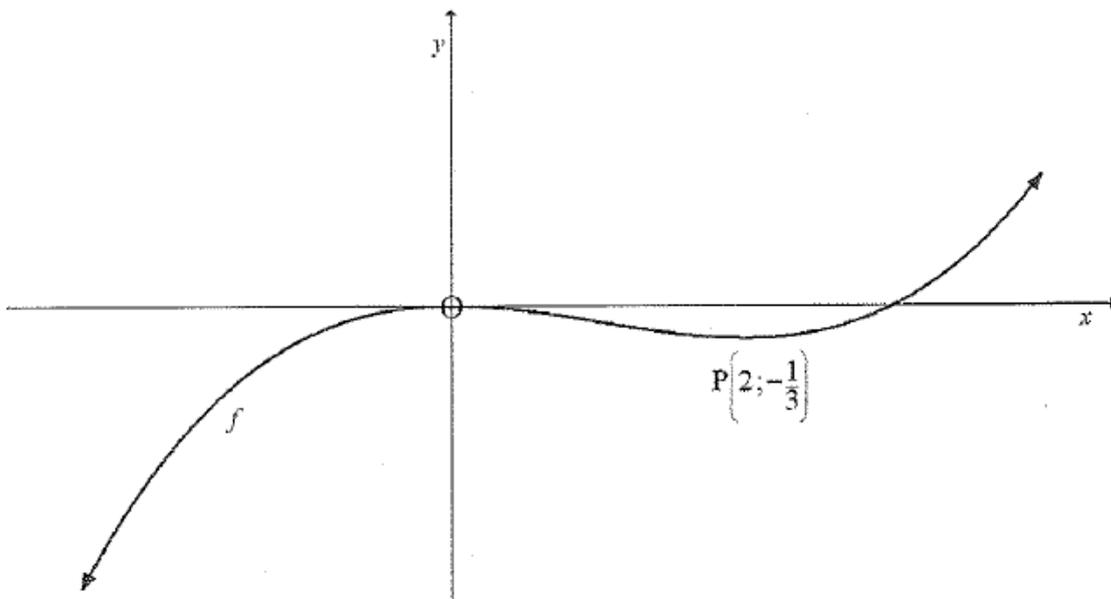


- 2.1 Calculate the coordinates of C if $B(1; 0)$ (4)
- 2.2 Determine the coordinates of E. (5)
- 2.3 For which values of x will f be concave up? (2)
- 2.4 Calculate the length of OG if the tangent to the curve at T is parallel to the tangent to the curve at D. (5)
- 2.5 Determine the value of m if $y = mx + c$ intersects f perpendicularly at $x = 5$. (3)

[19]

QUESTION 3: NORTH WEST

The graph of $f(x) = ax^3 + bx^2$ has stationary points at $P\left(2; -\frac{1}{3}\right)$ and $O(0; 0)$.

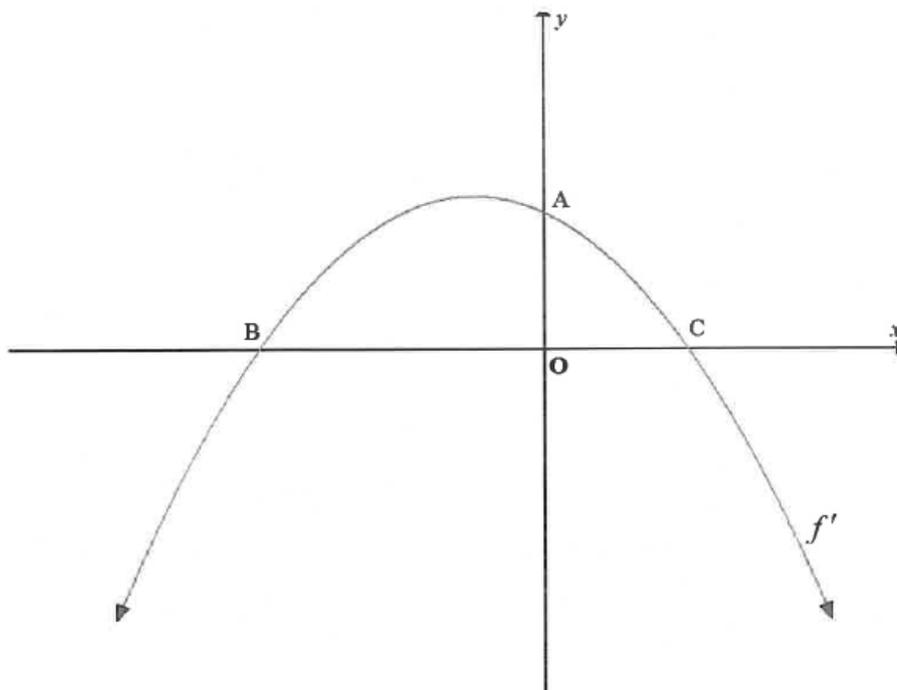


- 3.1 Prove that $a = \frac{1}{12}$ and $b = -\frac{1}{4}$ (4)
- 3.2 For which values of x will f be concave down? (3)
- 3.3 Determine the equation of the tangent to f at $x = -2$ in the form $T_n = mx + n$. (4)
- 3.4 Use the graph to determine the value of k for which $\frac{1}{12}x^3 - \frac{1}{4}x^2 - k = 0$ will have three real roots. (2)

[13]

QUESTION 4: GAUTENG

- 4.1 Sketched below is the graph of f' . The derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$. Points A, B and C are the intercepts of f' with the axes.



- 4.1.1 Write down the coordinates of A. (1)
- 4.1.2 Determine the coordinates of B and C. (3)
- 4.1.3 Which points on the graph of f will have exactly the SAME x -values as B and C? (1)
- 4.1.4 For which values of x will f be increasing? (3)
- 4.1.5 Determine the y -coordinate of the point of inflection of f . (4)
- 4.2 The tangent at $P(3; -10)$, to the curve is given by: $y = -x^2 - 1$, intersects the x -axis at point R. Line PT is drawn perpendicular to the x -axis with T on the x -axis. Determine the length of RT. (5)

[17]

QUESTION 5: FREE STATE

Given: $f(x) = x^3 - 12x - 16$

5.1 Calculate the:

5.1.1 Coordinates of the turning points of f . (5)

5.1.2 x -intercepts of f . (3)

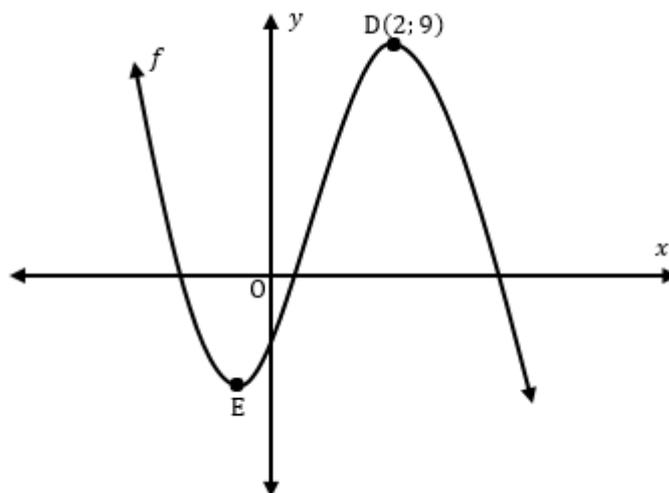
5.2 $y = 15x + p$ is a tangent to the graph of f . Calculate the x -coordinate of the point(s) of contact. (4)

5.3 For which value(s) of x will the given function be concave up? (3)

[15]

QUESTION 6: EASTERN CAPE

The diagram below shows the graph of $f(x) = -2x^3 + ax^2 + bx - 3$. $D(2; 9)$ and E are the turning points of f .



6.1 Determine the values of a and b . (5)

6.2 If $f(x) = -2x^3 + 5x^2 + 4x - 3$, calculate the coordinate of E . (3)

6.3 Determine the values of x for which:

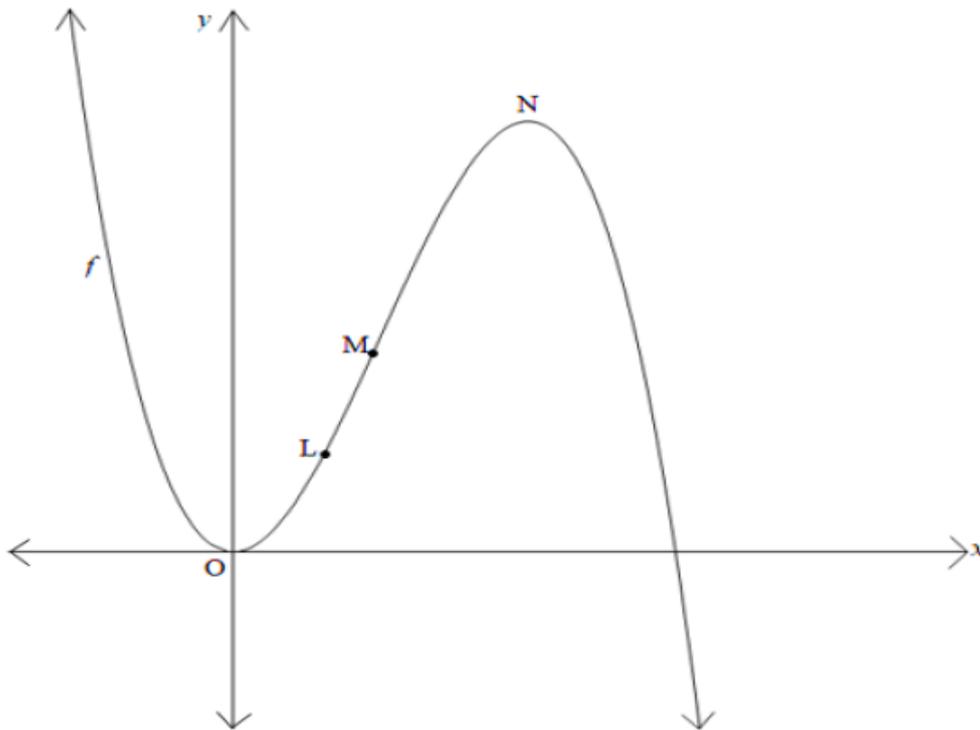
6.3.1 $f'(x) < 0$ (2)

6.3.2 The graph of f is concave down (3)

Determine the equation of the tangent to the graph of f at $P(-1; 0)$, in the form $y = mx + c$. (4)

QUESTION 7: KWA-ZULU NATAL

The diagram below shows the graph of $f(x) = ax^3 + bx^2$. The x -coordinates of L and M, are 1 and 2 respectively. The average gradient of f between L and M is 5,5. The equation of the tangent to the curve of f at $x = 6$ is $y = -18x + c$



- 7.1 Show that $a = -\frac{1}{2}$ and $b = 3$ (5)
- 7.2 The equation $f(x) = -\frac{1}{2}x^3 + 3x^2$ is given. Determine the coordinates of N, the turning point of f . (4)
- 7.3 The graph of f is concave up for $x < k$. Calculate the value of k . (3)

[12]

Part 3: Optimisation

QUESTION 1: LIMPOPO

A boy uses 12m of wire to form a circle and a square. Each side of the square is x m long.

- 1.1 Show that the total area of the square and the circle formed is given by: (6)

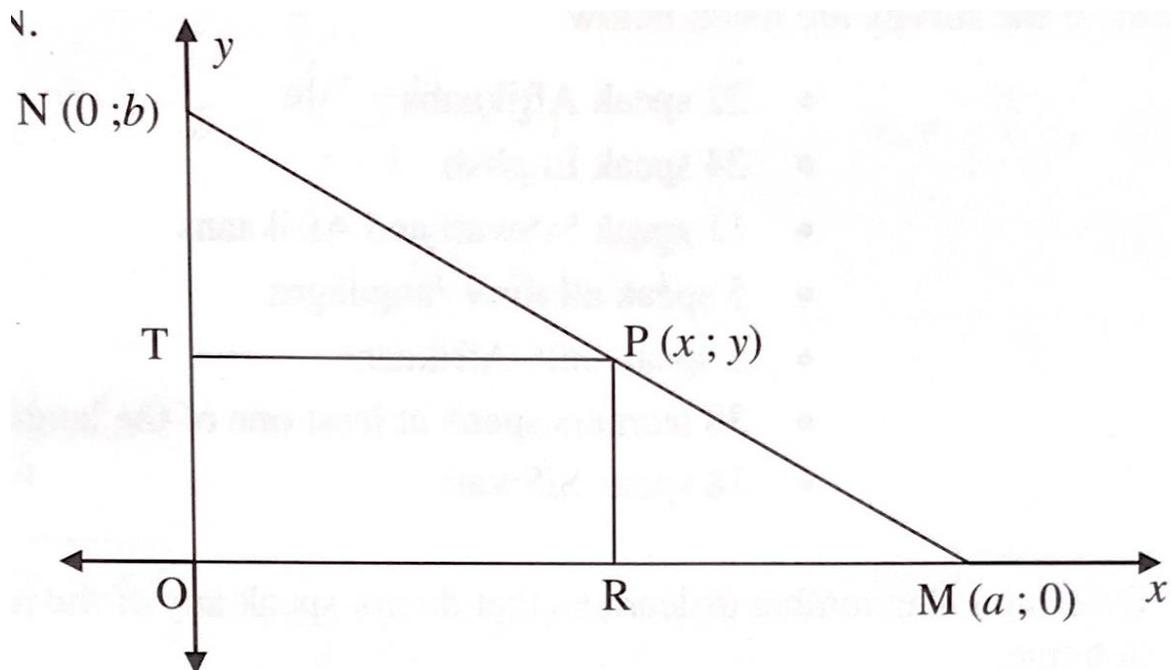
$$A = \frac{(4 + \pi)x^2 - 24x + 36}{\pi}$$

- 1.2 Determine the values of x for which the total area is minimum. (2)

[8]

QUESTION 2: MPUMALANGA

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let $OM = a$, $ON = b$ and $P(x; y)$ be any point on MN



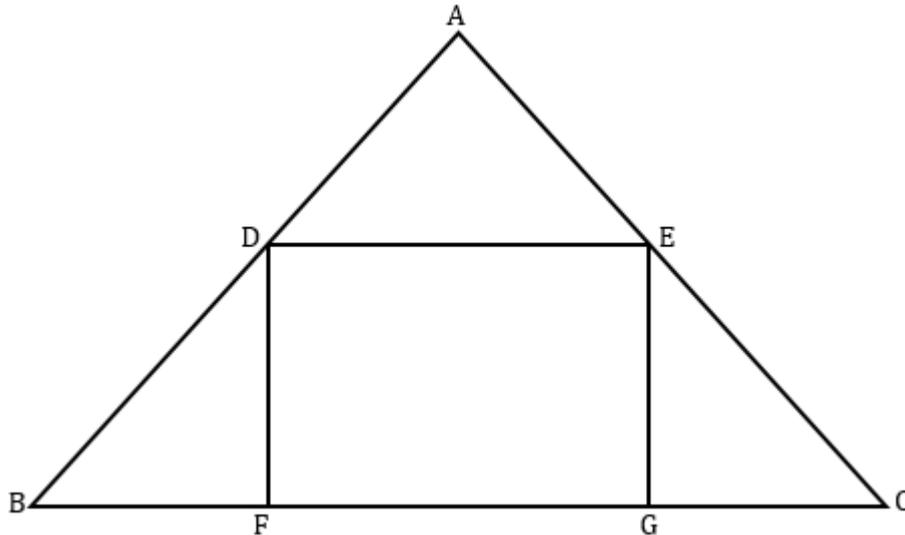
- 2.1 Determine an equation for MN in terms of a and b . (2)

- 2.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

[8]

QUESTION 3: NORTH WEST

In the sketch below, $\triangle ABC$ is an equilateral triangle with side $AB = m$ units. $DEGF$ is a rectangle with $BF = GC = x$ units.



3.1 Prove that the area of the rectangle $DEGF$ is $\sqrt{3}x(m - 2x)$. (3)

3.2 Determine, in terms of m , the maximum area of the rectangle. (4)

[7]

QUESTION 4: GAUTENG

The profit, $W(x)$, earned by a company to manufacture x bicycles per week, is given by:

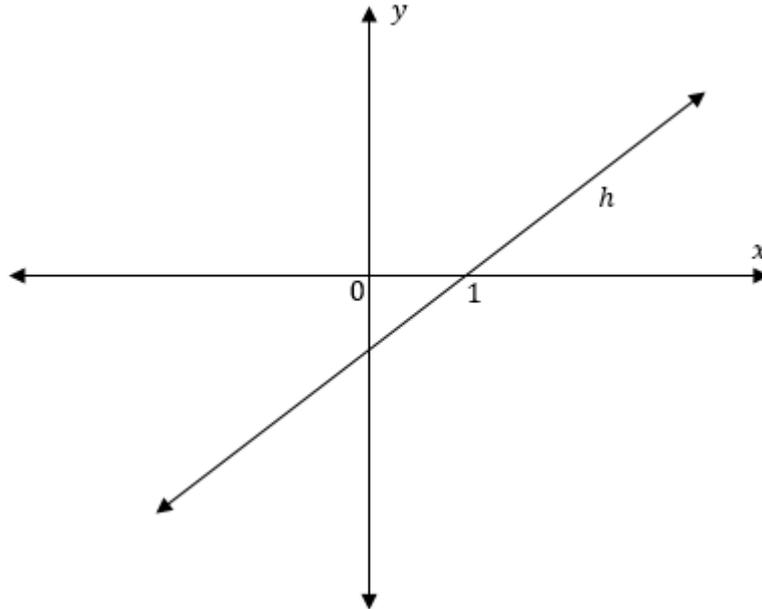
$$W(x) = -\frac{x^3}{150} + 3x^2 - 250x - 2\,700$$

What should the weekly production be to maximise profit?

[8]

QUESTION 5: FREE STATE

- 5.1 The diagram shows the straight line h , where $h(x) = f'(x)$.
The x -intercept of h is 1.
The following is true for function f : $f(1) = -3$ and $f(3) = 0$.



Draw a sketch graph of the function f , clearly indicating all x -intercepts and turning point(s). (3)

- 5.2 During an experiment the temperature, T in $^{\circ}\text{C}$ varies with time t in seconds, to the equation $T(t) = 60 + 27t - t^3$, $t \in [0; 6]$.

Calculate:

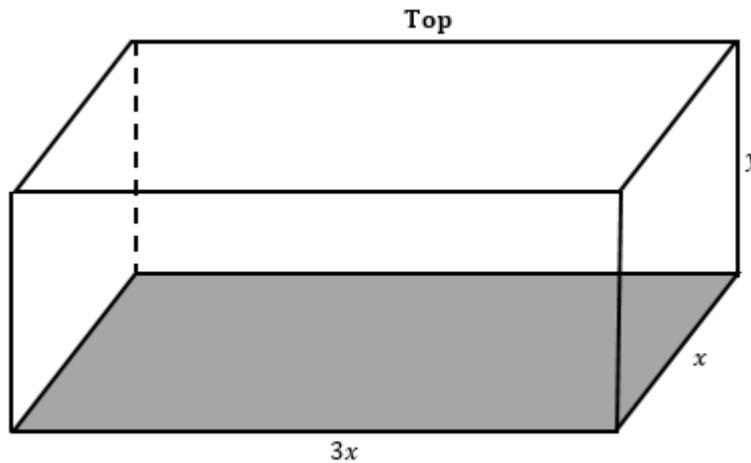
- 5.2.1 The average change of temperature between 3 and 6 seconds. (3)
- 5.2.2 After how many seconds the temperature will be a maximum. (3)

[9]

QUESTION 6: EASTERN CAPE

The wooden box in the diagram is a rectangular prism and it is open at the top. The dimensions of the base are $3x$ metres by x metres and the height is y metres.

The total surface area is 147 m^2 .



6.1 Show that: $y = \frac{147 - 3x^2}{8x}$ (2)

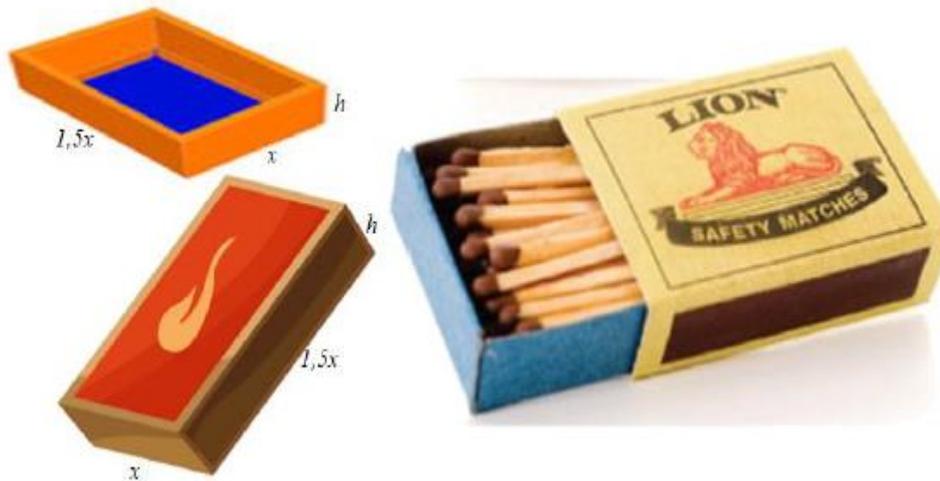
6.2 Calculate the value of x for which the volume of the box is a maximum. (5)

[7]

QUESTION 7: KWA-ZULU NATAL

A match box consists of an outer cover, open at both ends, into which slides a rectangular box without a top.

- The length of the box is one and a half times its breadth.
- The thickness of the material is negligible.
- The volume of the box is 25 cm^3 .



If the breadth of the box is x cm:

7.1 Write down the height of the box in terms of x . (2)

7.2 Show that the amount of the material used for the box is given by: (2)

$$A = \frac{100}{x} + 4,5x^2 + \frac{50}{1,5x}$$

7.3 Hence determine the breadth of the box such that the material used is a minimum. (3)

[7]

FINANCIAL MATHEMATICS

QUESTION 1: LIMPOPO

- 1.1 Daniel deposits a sum of money into a savings account. The bank offers interest at a rate of 5% p.a., compounded monthly. Calculate the effective interest rate that Daniel receives. (2)
- 1.2 A bank granted Matome a home loan for R800 000 when he started working some years ago. The loan was to be paid over a period of 25 years. The rate of interest on the loan was 10,75% p.a, compounded monthly.
- 1.2.1 Matome made his first repayment one month after the loan was granted and he continued to make monthly repayments thereafter. Calculate Matome's monthly repayments. (4)
- 1.2.2 Matome continued to make monthly repayments. Calculate the outstanding balance immediately after making the 240th repayment. (3)
- 1.2.3 After making payments for the full 20 years, Matome lost his job and missed the next six monthly repayments. He then found another job and resumed his monthly repayments. However, Matome started paying R9 000 monthly instead of the required amount. (5)

Did Matome settle the loan in the original repayment period? Justify your answer with relevant calculations.

[14]

QUESTION 2: MPUMALANGA

- 2.1 Jane deposits R x into an investment account. How long will it take for the value of the investment to double if the interest rate is 5,4% p.a., compounded monthly annually? (3)
- 2.2 Thabo starts printing company and needs to borrow money for start-up costs. He can make equal monthly payments of R3 300. What amount can Thabo borrow if the interest rate on the loan is 12% p.a. compounded monthly and the loan is granted over 5 years? (4)
- 2.3 A group of investor consider investing in a fund that promises growth at a rate of 5% p.a. compounded quarterly. Calculate the effective annual percentage rate of the growth promised. (3)
- 2.4 Sarah is 18 years old and wishes to accumulate R10 000 000 by the month before her 50th birthday. She will deposit equal monthly payments into an account 15% p.a. compounded monthly. The first payment starts on her 18th birthday and the last payment one month before her 50th birthday. (3)

Calculate the monthly instalment that Sarah will make.

[13]

QUESTION 3: NORTH WEST

3.1 Convert an effective interest rate of 11,3% p.a. to its equivalent rate per annum, compounded quarterly. (3)

3.2 Lisa opened a savings account and deposited R10 000 immediately into the account. The account paid interest at 5,3% per annum, compounded monthly. She started making additional monthly deposits of R500 into the account three months after the account was opened. Her last monthly deposits of R500 was made 5 years after the account was opened. (5)

How much was in the account 5 years after the account was opened?

3.3 Sam wants to buy a house and takes out a loan of R860 000. He can only afford to pay R7 200 per month starting one month after the loan is granted. The interest rate is compounded monthly at 9,5% p.a.

3.3.1 Calculate the number of payments that Sam will make to repay the loan? (4)

3.3.2 How much will Sam pay in the last month to settle the loan? (5)

[17]

QUESTION 4: GAUTENG

4.1 A survey conducted in December 2015 determined that 5,7 million South Africans were living with HIV. The researchers used a model of exponential growth $A = P(1 + i)^n$ to predict that there will be 6 million people living with HIV in December 2022. (3)

Calculate, as a percentage, the annual rate of increase that the researchers used for the 7 years.

4.2 Shimmy invests R4 000 000 into an account earning interest of 6% per annum, compounded monthly. She withdraws R30 000 per month. Her first withdrawal is exactly one month after she deposited the R4 000 000.

4.2.1 How many withdrawals of R30 000 will Shimmy be able to make? (5)

4.2.2 How many withdrawals will Shimmy be able to make if she changes the amount to 20 000? Substantiate your answer. (3)

4.3 Estrid opened a savings account with a single deposit of R1 000 on 1 April 2022. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2022 and her last payment on (4)

30 September 2023. The account earns interest at 15% per annum, compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (on 20 September 2023).

[15]

QUESTION 5: FREE STATE

- 5.1 How many years will it take to triple an investment if the interest is compounded annually at a rate of 9,8% p.a.? (3)
- 5.2 Andile needs R64 000 for a holiday. He started to invest a fixed amount of his salary at a rate of 8,5% p.a. compounded monthly, at the end of each month, for ten years.
- 5.2.1 Calculate the monthly payment he will have to invest to achieve this. (3)
- 5.2.2 If Andile has stopped his payment at the end of eight years, what will the total of his investment be at the end of ten years? (3)
- 5.3 Madri took out a loan of R400 000 at an interest rate of 10,4% p.a., compounded monthly. She repaid the loan at the end of the first month and every month for 15 years. Her monthly instalment is R4 396,83.
- 5.3.1 Calculate the outstanding balance after nine years. (3)
- 5.3.2 How much interest did she pay over the nine years? (3)

[15]

QUESTION 6: EASTERN CAPE

- 6.1 Lufezo deposited R97 000 into an account that offered at 9,1% p.a. compounded quarterly. Calculate how many years it took for an investment to reach R166 433. (4)
- 6.2 On 1 January 2018 a school bought a new bus for R482 000. On that day they also started a sinking fund to make provisions for a new bus in 5 years' time.
- 6.2.1 Over the next 5 years the value of the bus depreciates at 14,7% p.a. on the reducing balance method. Calculate the trade-in value of the new bus in 5 years' time. (2)
- 6.2.2 The price of these buses increases by 8,1 % per year. Calculate the price of a new bus on 1 January 2023, i.e. after 5 years. (2)

- 6.2.3 The bank offered an interest rate of 7,3% p.a., compounded monthly, (6)
for the sinking fund. The first payment, x rands, was made in the
fund on 1 January 2018 and thereafter the same amount was
deposited on the first day of every month. The last payment was
made on 1 December 2022.

On 31 December 2022 the school bought a new bus and used the
trade-in value of the old bus as a deposit.

Calculate the monthly payment into the sinking fund.

[14]

QUESTION 7: KWA-ZULU NATAL

- 7.1 Minenhle bought a laptop for R15 800. The laptop depreciates at 12% p.a. on a reducing balance. After how many years will the value of his laptop be R10 767,26? (3)
- 7.2 Calculate the effective yearly interest rate if an investment offers a nominal interest rate of 7,64% p.a. compounded half-yearly. (2)
- 7.3 Mandla is planning to buy a car in 2 years' time. He invests 500 into a savings account at the end of each month for 2 years, starting in one month's time so that he can use the money as a deposit for his car. Interest is calculated at 5,8% p.a. compounded monthly.
- 7.3.1 How much money will be in his savings account at the end of 2 years? (3)
- 7.3.2 The cost of the car is R368 400. Mandla uses all the money from his savings account as the deposit for his new car. He takes out a loan to pay for the balance after paying the deposit. Interest is calculated at 10,4% p.a. compounded monthly. Calculate Mandla's monthly instalments if the car must be paid for in 6 years. (4)
- 7.3.3 Calculate the outstanding balance immediately after paying the 56th instalment. (3)

[15]

PROBABILITY

QUESTION 1: LIMPOPO

- 1.1 At a school, learners may choose subjects from only one of the following streams: Commerce, Science or Humanities. There are 100 learners in Grade 12 at this school. Their choice of subjects is summarised in the table below.

	Commerce (C)	Science (S)	Humanities (H)	Total
Boy (B)	12	15	23	50
Girl (G)	8	20	22	50
Total	20	35	45	100

- 1.1.1 Calculate the probability that a learner chosen at random from this group is a girl. (1)
- 1.1.2 Calculate the probability that a learner chosen at random from this group has chosen Science. (1)
- 1.1.3 Are the events: being a girl and choosing Sciences independent: Justify your answer. (3)
- 1.2 How many four-digit codes will be formed using digits: 0; 1; 2; 3; 4; 7; 9? Consider the following conditions: (2)
- The code must be an even number (i.e. must be divisible by 2)
 - The first digit must be less than 4.
 - Digits may be repeated.
- 1.3 The letters of the word LEOPARD are randomly arranged to into different 7 – letter words. Letters may not be repeated.
- 1.3.1 In how many different ways can the letters of this word be arranged? (1)
- 1.3.2 Determine the number of different ways in which a word formed will start with a vowel and end with a consonant. (3)
- 1.3.3 Calculate the probability that in the different words formed in 1.3.1, all vowels will not follow each other. (4)

[15]

QUESTION 2: MPUMALANGA

2.1 A language survey was done with 45 grade 12 learners in Mbombela Secondary School. The aim of the study was to find out what language learners speak in their homes. The results are listed below:

- 22 speak Afrikaans
- 24 speak English
- 11 speak SiSwati and Afrikaans
- 5 speaks all three languages
- 3 speak only Afrikaans
- 38 learners speak at least on the languages at home
- 18 speak SiSwati

2.1.1 Determine the number of learners that do not speak any of the named languages at home. (1)

2.1.2 If x learners speak English and SiSwati but not Afrikaans, draw a Venn diagram to represent the information. (4)

2.1.3 Solve for x . (2)

2.1.4 What is the probability that a learner speaks only two of the three languages at home? (2)

2.2 Three brothers Owen, Richard and Robert are to run a race, which has 8 runners in total. The eight competitors line up one to a lane in the lanes numbered 1 to 8.

2.2.1 Write down the total number of possible arrangements of the runners at the starting line. (1)

2.2.2 Calculate the total number of arrangements in which the three brothers are all next to each other. (3)

2.2.3 Calculate the probability that Owen is in lane 1, Robert is in lane 2 and Richard is in lane 3. (2)

[15]

QUESTION 3: NORTH WEST

3.1 Given: $P(A) = 0,3$, $P(B) = 0,43$ and $P(C) = 0,18$.

3.1.1 What is the probability that A and C will happen simultaneously? (1)

3.1.2 What is the probability that at least one of A or B will take place? (3)

3.2 A group of 250 Grade 12 learners participated in a survey. They were asked if they used the social media app TikTok. The results are represented in the table below.

	Use TikTok	Don't use TikTok	Total
Girls	105	25	130
Boys	68	52	120
Total	173	77	250

3.2.1 A name is drawn randomly out of the group. What is the probability that the name drawn will be a girl and using TikTok? (1)

3.2.2 Are the events of being a girl and using TikTok independent? Motivate your answer by doing appropriate calculations. (4)

3.3 A group of 5 boys, 6 girls and a teacher goes to the movies. They buy 12 tickets in a row next to one another. One of the seats is next to an aisle.

3.3.1 In how many different ways can they be seated? (1)

3.3.2 To avoid disruptive behaviour, the teacher considers to place the following restriction on how they may sit: (3)

The teacher at the end next to the aisle, and the children in any order as long as Peter and John do not sit together.

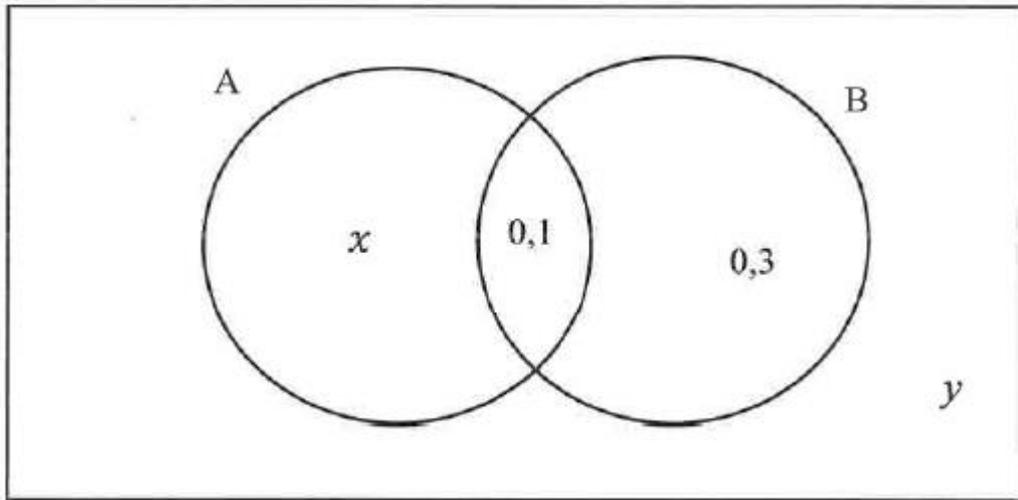
Calculate in how many different ways they be seated.

3.3.3 Duncan wants to sit next to Suzie. If the whole group of twelve are seated at random, What is the probability that the two of them will sit together? (3)

[16]

QUESTION 4: GAUTENG

- 4.1 Machine A and machine B are two different coin-pressing machines that operate at the same time. The probability that machine A ONLY presses a R5 coin, is x and the probability that machine B Only presses a R5 coin, is 0,3. The probability that both the machines press R5 coins at the same time is 0,1.



- 4.1.1 If A and B are independent events, determine the values of x and y . (4)
- 4.1.2 Determine the probability that exactly one of the machines is pressing a R5 coin. (1)
- 4.2 Wilson takes a driver's test. The probability that he will succeed on his first attempt is $\frac{3}{7}$.
- For each attempt that he redoes the test, the probability of passing increases to $\frac{3}{5}$.
- 4.2.1 What is the probability that Wilson will succeed after 2 attempts? (2)
- 4.2.2 Determine the probability that Wilson will succeed after 3 attempts. (2)

[9]

QUESTION 5: GAUTENG

- 5.1 When Marge turned eight, her friends Emily, Klara, Cory, Shirley and Penny were invited to her birthday party. Marge and her friends sat in a row and played a game. In how many ways can they be seated if:
- 5.1.1 They sit in alphabetical order? (1)
- 5.1.2 Emily and Klara do NOT want to sit next to each other? (3)
- 5.2 The probability that a certain rugby team has all its players fit to play is 70%. The probability that they will win a game if all the players are fit is 90%. When they are not fit, the probability of them winning becomes 45%/
Calculate the probability of them winning the FIRST game. (2)
- [6]

QUESTION 6: FREE STATE

- 6.1 Given: $P(A) = 0,4$ and $P(B) = 0,5$
- 6.1.1 Calculate $P(A \text{ or } B)$ if A and B are mutually exclusive events. (2)
- 6.1.2 Calculate $P(A \text{ or } B)$ if A and B are independent events. (3)
- 6.2 A four-digit code must be set using the letters A, E, I, O, U and digits 0 to 9. The letters may be repeated, but the digits may not be repeated. The code must consist of two letters and two digits in that order, for example, UO19.
- 6.2.1 How many different codes are possible with the information given? (4)
- 6.2.2 What is the probability that a code that is picked randomly will start with and be an even number? (4)
- [13]

QUESTION 7: EASTERN CAPE

- 7.1 A survey was carried out among 210 people to determine whether they prefer watching rugby or soccer on TV. The results are shown in the contingency table below.

	WATCH SOCCER	WATCH RUGBY	TOTAL
Female	72	a	120
Male	54	36	90
Total	b	84	210

- 7.1.1 Determine the values of a and b . (2)
- 7.1.2 Give the probability that an individual chosen at random is a female preferring to watch soccer. (2)
- 7.1.3 Are the events 'being male' and 'watch rugby' independent? Justify your answer with calculations. (4)

Example:

A	B	C	1	2	3
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- 7.2 The password of a computer consists of 3 letters and 3 digits, in that order. All 10 digits and 26 letters of the alphabet may be used, without repetition.
- 7.2.1 How many different passwords can be formed out of the 10 digits and 26 letters? (2)
- 7.2.2 Calculate the probability that the first letter of a password formed is a vowel and the last digit of the password is a factor of 9. (4)

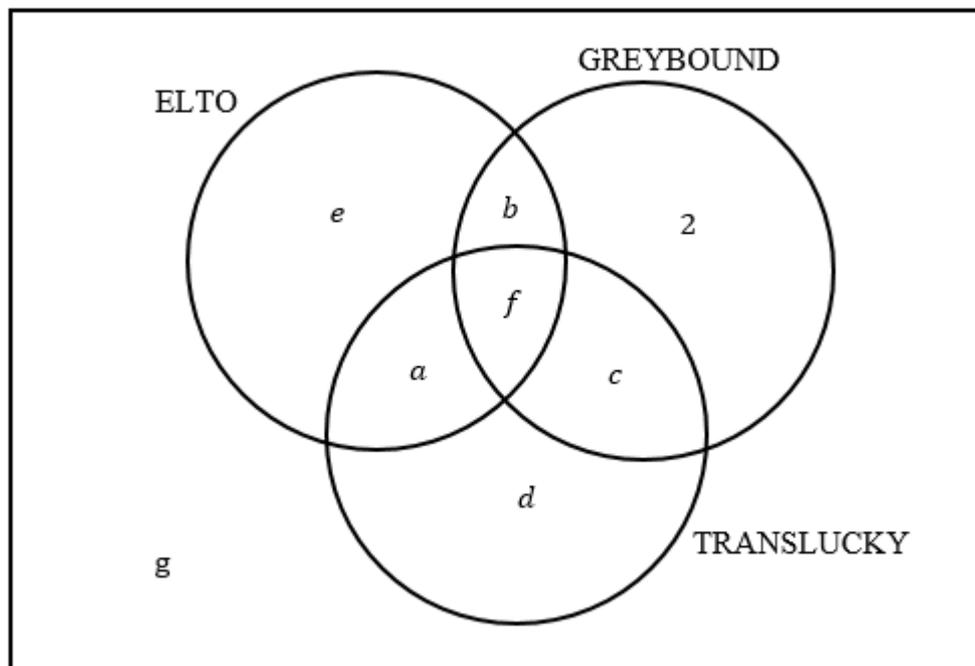
[14]

QUESTION 8: KWA-ZULU NATAL

A group of 40 people was asked which bus company they liked to travel by: Elto, Greybound or Translucky bus company.

- 3 people liked travelling by all bus companies.
- 7 people liked to travel by Translucky and Elto.
- 3 people did not like using any of the bus companies.
- The probability of a randomly selected person liking to travel by Greybound and Elto is $\frac{2}{5}$.
- The probability of a randomly selected person liking to travel by Greybound and Translucky is $\frac{1}{5}$.
- The probability of a randomly selected person liking to travel by Translucky only is $\frac{1}{10}$.
- The probability of a randomly selected person liking to travel by Elto is $\frac{13}{20}$.

The partially completed Venn diagram drawn below represents the given information.



- 8.1 Use the given information to determine the values of a , b , c , d and e . (5)
- 8.2 How many people liked to travel by Greyhound bus company? (1)
- 8.3 Calculate the probability of a randomly selected person liking to travel by only one bus company. (2)

[8]

QUESTION 9: KWA-ZULU NATAL

Nonhle who is a Grade 12 learner has 8 textbooks from different subject: Mathematics, English, Accounting, History, Tourism, Afrikaans, Geography and Drama which she wants to arrange in a line on a shelf.

- 9.1 In how many ways can the textbooks be arranged? (1)
- 9.2 In how many ways can the textbooks be arranged if the Mathematics textbook and the accounting textbook must be on each end of the shelf? (3)
- 9.3 If the Mathematics textbook and the Accounting textbook must be on each end of the shelf, what is the probability that the History textbook and the Tourism textbook are not next to each other? (4)

[8]